

## THE ROLE OF ENVIRONMENTAL FACTORS IN GROWTH ACCOUNTING

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### SUMMARY

This paper explores a relatively new methodology, the directional distance function method, to analyse productivity growth. The method allows us to explicitly evaluate the role that undesirable outputs of the economy, such as carbon dioxide and other greenhouse gases, have on the frontier production process which we specify as a piecewise linear and convex boundary function. We decompose productivity growth into efficiency change (catching up) and technology change (innovation). We test the statistical significance of the estimates using recently developed bootstrap methods. We also explore implications for growth of total factor productivity in the OECD and Asian economies. Copyright © 2004 John Wiley & Sons, Ltd.

### 1. INTRODUCTION

In traditional productivity analysis environmental byproducts of the production or development process are ignored and, as such, are assumed to be freely disposable. Using a recently developed technique, the directional distance method, we analyse the impact that explicit treatment of the external costs of carbon dioxide has on the productivity growth of OECD and Asian economies. We examine how the econometric developments may affect our comfort in and use of productivity forecasts for growth in the USA, OECD and in Asia. Our paper speaks to the international debate on trade-offs between growth and environmental protection.

We decompose productivity growth into changes in technical efficiency over time and shifts in technology. These allow us to identify the major factors in each country's growth process. We do not observe the true production frontier but construct it from our sample, and we provide a statistical interpretation of the indices via recently developed bootstrap methods introduced by Simar and Wilson (1998, 1999, 2000a, b).

Radial technical efficiency measures were first developed by Farrell (1957). Caves *et al.* (1982) defined the input-based Malmquist productivity index as the ratio of two input distance functions while assuming no technical inefficiency in the sense of Farrell (1957). Färe *et al.* (1994) extended the Caves *et al.* approach by dropping the assumption of no technical inefficiency and developed a Malmquist index of productivity that could be decomposed into indices describing changes in technology and efficiency. This approach has been used widely. These indices have been used to study issues ranging from deregulatory dynamics in the US airline industry (Alam and Sickles, 2000) to the convergence of per capita incomes of the OECD countries (Färe *et al.*, 1994).

Chung *et al.* (1997) introduce a directional distance function approach, the Malmquist–Luenberger index, to analyse models of joint production of goods and bads. Although these terms may

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be pejorative, they provide a convenient delineation between the goods and services that trade in a formal market and those for which the formal market has not yet been established and are generally viewed as having an unallocated cost. The method credits firms for reductions in bads and increases in goods. The Malmquist index can also be applied to the undesirable output case by modifying the direction in which the goods and bads are traded off. Boyd *et al.* (1999) have recently analysed OECD countries assumed to possess a two-output/two-input technology using deterministic Malmquist and Malmquist–Luenberger indices.

We apply Malmquist and Malmquist–Luenberger index methods to a sample of OECD and Asian countries that are assumed to possess a two-output/three-input technology over the period 1980–1990 and 1980–1995, respectively. We analyse how productivity growth is affected by lifting the free disposability assumption and test the statistical significance of the indices of productivity growth using newly developed bootstrap methods. Historically, the growth in an economy has been due to the growth of inputs and growth in the productivity of those inputs. Factors that influence the latter will influence wealth creation as well as the ability of the economy to maintain wealth levels as it reallocates resources to pay for pollution abatement.

The paper is organized as follows. We begin with a review of the distance functions and productivity index models in Section 2. This is followed by a discussion of the bootstrapping algorithm in Section 3. Section 4 contains a discussion of data and results. Section 5 concludes.

## 2. THE PRODUCTIVITY INDICES

To define the output-based productivity index, we assume that the production technology  $F^t$  for each time period  $t = 1, \dots, T$  transforms the inputs  $x^t \in R_+^L$  into outputs, goods  $y^t \in R_+^M$  and bads  $b^t \in R_+^N$ :

$$F^t = \{(x^t, y^t, b^t) | x^t \text{ can produce } (y^t, b^t)\} \quad (1)$$

The production technology consists of the set of all feasible input and output vectors. In order to address the fact that the reduction of bad outputs is costly, we impose weak disposability of outputs, i.e.

$$(x^t, y^t, b^t) \in F^t \quad \text{and} \quad 0 \leq \theta \leq 1 \quad \text{imply} \quad (x^t, \theta y^t, \theta b^t) \in F^t \quad (2)$$

Thus a reduction of undesirable outputs can be attained by the reduction of goods, given fixed input levels. Clearly, if undesirable outputs could be disposed of freely, we could reduce only undesirable outputs. The production technology also is assumed to produce both desirable and undesirable outputs, and it is assumed that it cannot produce one without the other, i.e.

$$\text{if } (x^t, y^t, b^t) \in F^t \quad \text{and} \quad b^t = 0 \quad \text{then} \quad y^t = 0$$

### 2.1. The Malmquist Productivity Index

Caves *et al.* (1982) defined the Malmquist productivity index as the ratio of two distance functions. However, they encountered computational complications in calculating the distance function directly and were forced to approximate the Malmquist productivity index with the discrete approximation to the Divisia index, the Törnqvist index. The conditions under which the use of the Törnqvist index was justified are rather strong, in particular the firm is assumed to be technically efficient. Färe *et al.* (1995, 1998) provided a method to calculate the index directly in

the presence of technical inefficiency by noting that the output distance function and the radial output-based technical efficiency measure constructed from nonparametric frontier methods were inversely related. Following Shephard (1970), the output distance function at time  $t$  is written as:

$$\begin{aligned} D_0^t(x^t, y^t, b^t) &= \inf\{\theta | (x^t, y^t/\theta, b^t/\theta) \in F^t\} \\ &= (\sup\{\theta | (x^t, \theta y^t, \theta b^t) \in F^t\})^{-1} \end{aligned} \quad (3)$$

where superscript  $t$  of the distance function denotes the time of production. By construction  $D_0^t(x^t, y^t, b^t) \leq 1$  if and only if  $(x^t, y^t, b^t) \in F^t$ . When  $D_0^t(x^t, y^t, b^t) = 1$  the country is on the boundary of the production set and thus is employing the frontier technology.

To construct the Malmquist productivity index one first needs to specify the distance function with respect to the two adjacent time periods wherein the technology is in place and the resource allocation decisions are made:

$$D_0^t(x^{t+1}, y^{t+1}, b^{t+1}) = \inf\{\theta | (x^{t+1}, y^{t+1}/\theta, b^{t+1}/\theta) \in F^t\} \quad (4)$$

The index measures the maximum proportional change of outputs required to produce  $(y^{t+1}, b^{t+1})$  at the technology level in place at time  $t$ . This may not be feasible if the combination of the outputs, say  $y^{t+1}, b^{t+1}$  for a single desirable and undesirable output, is not on the hyperplane generated from outputs at time  $t$ .

The output-based Malmquist productivity change index (Malmquist, 1953) is defined as:

$$M_0^{t,t+1} = \left( \frac{D_0^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^t(x^t, y^t, b^t)} \cdot \frac{D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^{t+1}(x^t, y^t, b^t)} \right)^{1/2} \quad (5)$$

and is the geometric mean of the two output distance functions' ratios with respect to time  $t$  and  $t + 1$ . The Malmquist index is the most widely used output index and is particularly attractive for our purposes since it does not rely on prices, specifically the price of CO<sub>2</sub>, in order to construct it. It is based on the ratio of two output distance functions defined for period  $t$  and period  $t + 1$  technologies. If the distance functions for period  $t$  and  $t + 1$  technologies are translog, then the geometric mean of two Malmquist output indices using period  $t$  and  $t + 1$  technologies and period  $t$  and  $t + 1$  input vectors is equivalent to the Törnqvist index which is flexible and superlative. If the distance functions are quadratic then the geometric mean of the two Malmquist output indices is equivalent to the Fisher (1922) ideal index, which is the geometric mean of the Laspeyres and Paasche indices. The set of axiomatic properties such indices possess is substantial and the reason for their wide use in productivity measurement. The linear program used below constructs the distance functions as piecewise linear approximations to any convex function. The advantages of the Malmquist productivity index have been pointed out by Diewert (1981, 1983, 1992) and Balk (1995, 1997), and are discussed at length in Coelli *et al.* (1998).

We illustrate this formulation of the Malmquist index in Figure 1.<sup>1</sup> Assume that there are two best practice frontiers based on period  $t$  and  $t + 1$  data. Observed input and output data from period  $t + 1$  are above the period  $t$  best practice frontier and the period  $t$  data are below the period  $t + 1$  best practice frontier. This is consistent with positive productivity growth.

<sup>1</sup> This example can be found in Färe and Grosskopf (1996).

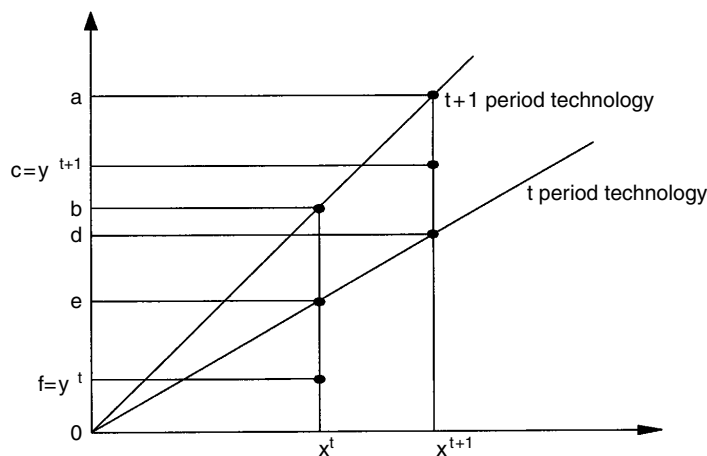


Figure 1. Malmquist productivity index

For this particular country the Malmquist index can be expressed as:

$$M_0^{t,t+1} = \left( \frac{0c/0a}{0f/0b} \frac{0c/0d}{0f/0e} \right)^{1/2}$$

or equivalently as:

$$M_0^{t,t+1} = \left( \frac{0c/0a}{0f/0e} \right) \left( \frac{0a/0d}{0e/0b} \right)^{1/2}$$

The first term is the change in efficiency between period  $t$  and  $t+1$  while the second term measures the shift in the frontier. This decomposition can be formalized by rewriting the general form of the output-based Malmquist productivity change index equivalently as:

$$M_0^{t,t+1} = \frac{D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^t(x^t, y^t, b^t)} \cdot \left( \frac{D_0^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})} \cdot \frac{D_0^t(x^t, y^t, b^t)}{D_0^{t+1}(x^t, y^t, b^t)} \right)^{1/2} \quad (6)$$

where the first term measures the change in relative efficiency between  $t$  and  $t+1$  (ECH), and the second term captures the shift in technology between the two periods (TCH). The decomposition of the Malmquist total factor productivity (TFP) index into a portion due to technological and efficiency change is based on a simple algebraic manipulation of the Malmquist output-oriented TFP index and is discussed in Färe *et al.* (1994) using nonparametric methods. Nishimizu and Page (1982) provide a parametric approach to the same problem. Using the Färe *et al.* (1994) approach we construct the distance functions, the Malmquist TFP index, and then manipulate it to represent two terms. The first term is the Farrell technical efficiency change between period  $t$  and  $t+1$ . Efficiency change is the ratio of Farrell technical efficiency in period  $t+1$  to Farrell technical efficiency in period  $t$ . The second term is the index of technical change. It is the geometric mean of technical change between period  $t$  and  $t+1$  using input vectors from the two periods.

We can thus write

$$\begin{aligned} \text{ECH} &= \frac{D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^t(x^t, y^t, b^t)} \\ \text{TCH} &= \left( \frac{D_0^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})} \cdot \frac{D_0^t(x^t, y^t, b^t)}{D_0^{t+1}(x^t, y^t, b^t)} \right)^{1/2} \end{aligned} \quad (7)$$

The standard Malmquist index assumes free disposability of undesirable outputs. Four different types of distance functions are needed by (5). The distance function of country  $k'$  at  $t$  will be constructed by the linear program:

$$\begin{aligned} (\hat{D}_0^t(x^t(k'), y^t(k'), b^t(k')))^{-1} &= \text{Max } \theta(k') & (8) \\ \text{Subject to } \theta(k') y_m^t(k') &\leq \sum_{k=1}^K z^t(k) y_m^t(k) \quad m = 1, \dots, M \\ \sum_{k=1}^K z^t(k) x_l^t(k) &\leq x_l^t(k') \quad l = 1, \dots, L \\ z^t(k) &\geq 0 \quad k = 1, \dots, K \end{aligned}$$

where  $M$ ,  $L$  and  $K$  are the number of desirable outputs, inputs and countries, respectively, while the  $z$ 's are intensity variables whose interpretation will be explained below. Construction of the distance function by linear programming methods is relatively intuitive. Let us assume as an example<sup>2</sup> that the technology is characterized by a two-output/one-input production process with no bads and that there are only three countries measured at time  $t$ . For  $k' = 1$  (country 1) the level of output at time  $t$  is  $y^t(1) = \{2, 1\}$  and its level of input is  $x^t(1) = \{1\}$ . For  $k' = 2$  (country 2) the level of output at time  $t$  is  $y^t(2) = \{1, 2\}$  and its level of input is  $x^t(2) = \{1\}$ . For  $k' = 3$  (country 3) the level of output at time  $t$  is  $y^t(3) = \{1, 1\}$  and its level of input is  $x^t(3) = \{1\}$ . The output–distance function for, say, country  $k' = 3$  at time  $t$  is constructed as:

$$\begin{aligned} \hat{D}_0^t(1, 1, 1)^{-1} &= \text{Max } \theta & (9) \\ \text{Subject to } \theta \cdot 1 &\leq z^t(1) \cdot 2 + z^t(2) \cdot 1 + z^t(3) \cdot 1 & (10) \\ \theta \cdot 1 &\leq z^t(1) \cdot 1 + z^t(2) \cdot 2 + z^t(3) \cdot 1 \\ 1 &\leq z^t(1) \cdot 1 + z^t(2) \cdot 1 + z^t(3) \cdot 1 \\ 0 &\leq z^t(1), z^t(2), z^t(3) \end{aligned}$$

The output possibility set for the constant returns to scale technology is characterized in Figure 2 as a piecewise linear and convex boundary function which for large  $K$  can approximate any smooth convex function. The solution is  $z^t(1) = z^t(2) = 1/2$ , and  $z^t(3) = 0$ . The Farrell radial technical efficiency score is 1.5, meaning that country 3 can increase both outputs by 50% if it operated on the boundary. This corresponds to a distance function value of 0.667. Countries 1 and 2 in

<sup>2</sup> This example also can be found in Färe and Grosskopf (1996).

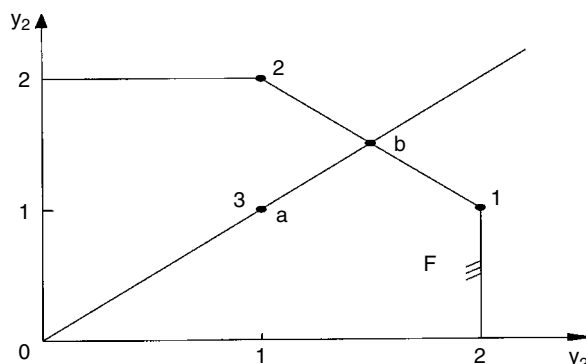


Figure 2. The distance function

this example are on the boundary and thus have distance function values of 1.0 and values of the intensity variables of  $\{1, 0, 0\}$  and  $\{0, 1, 0\}$ , respectively.

Since the free disposability of undesirable outputs is a rather strong assumption, especially in the context of environmental waste byproducts, we can define another type of Malmquist index that relaxes this assumption. A Malmquist index can be constructed by measuring the productivity change of desirable outputs while holding undesirable outputs constant. This may be the appropriate productivity measure when there are production quotas for undesirable outputs. The (more goods direction) distance function measures the relative distance to the highest feasible mix without changing the level of undesirable outputs. The four different distance functions needed to construct this index are  $D_0^t(x^t, y^t, b^t)$ ,  $D_0^{t+1}(x^t, y^t, b^t)$ ,  $D_0^t(x^{t+1}, y^{t+1}, b^{t+1})$  and  $D_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})$ . The formulations of the linear programs are in Appendix A.

## 2.2. The Malmquist–Luenberger Productivity Index

The Malmquist–Luenberger productivity index is based on the output-oriented directional distance function (Chung *et al.*, 1997). This is different from the Malmquist index which changes the desirable outputs and undesirable outputs proportionally because it chooses the direction to be  $g = (y^t, -b^t)$ , more good outputs and less bad outputs. The rationale of this kind of directional choice is that there might be institutional regulations limiting an increase in bad outputs, in particular pollutant emission. Figure 3 shows three different reference directions for each index. The direction vector labelled More Outputs is an example of output growth in which CO<sub>2</sub> production is viewed as a recognized concomitant of aggregate growth but it ignores the deleterious aspect of CO<sub>2</sub>, increasing it along with the production of goods and services that provide direct benefit to consumers. The direction vector labelled More Goods fixes the level of CO<sub>2</sub> production at current levels, similar to what is proposed in certain versions of the Kyoto negotiations that target levels of OECD production of CO<sub>2</sub> based on then current (1990 or so) levels. Finally, the direction vector labelled ML reduces the level of CO<sub>2</sub> emissions by the same amount as the increase in the production of marketable goods and services. This is a direction vector that is favoured by many in the environmental movement and can be viewed as the polar case to the More Outputs direction vector.

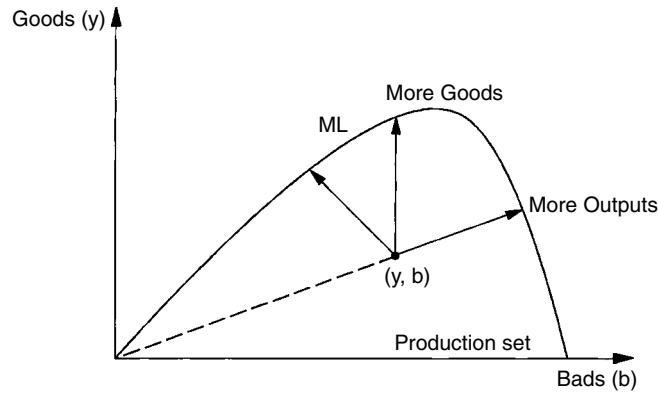


Figure 3. Distance functions

In order to develop a productivity index that accommodates these various directions for a movement to the frontier, we define the production technology in terms of the output set  $(y^t, b^t)$ :

$$P(x^t) = \{(y^t, b^t) | (x^t, y^t, b^t) \in F^t\} \tag{11}$$

and then define the directional distance function as:

$$\bar{D}_0^t(x^t, y^t, b^t; g) = \sup\{\beta | (y^t + \beta g_y, b^t - \beta g_b) \in P(x^t)\} \tag{12}$$

where  $g_y$  and  $g_b$  are subvectors for  $y^t$  and  $b^t$  of the direction vector  $g$ .

Chung *et al.* (1997) define the output-oriented Malmquist–Luenberger productivity index between periods  $t$  and  $t + 1$  as:

$$ML_0^{t,t+1} = \left( \frac{\{1 + \bar{D}_0^t(x^t, y^t, b^t; y^t, -b^t)\}}{\{1 + \bar{D}_0^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})\}} \times \frac{\{1 + \bar{D}_0^{t+1}(x^t, y^t, b^t; y^t, -b^t)\}}{\{1 + \bar{D}_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})\}} \right)^{1/2} \tag{13}$$

The Malmquist–Luenberger index coincides with the Malmquist index when the direction  $g$  is  $(y, b)$  rather than  $(y, -b)$ . The relationship between Shephard’s distance function and the directional distance function is:

$$\begin{aligned} \bar{D}_0^t(x^t, y^t, b^t; y, b) &= \sup\{\beta | (y^t + \beta g_y, b^t + \beta g_b) \in P(x^t)\} \\ &= \sup\{\beta | (y^t(1 + \beta), b^t(1 + \beta)) \in P(x^t)\} \\ &= \sup\{-1 + (1 + \beta) | (y^t(1 + \beta), b^t(1 + \beta)) \in P(x^t)\} \\ &= -1 + \sup\{(1 + \beta) | (y^t(1 + \beta), b^t(1 + \beta)) \in P(x^t)\} \\ &= -1 + \frac{1}{D_0^t(x^t, y^t, b^t)} \end{aligned}$$

The Malmquist–Luenberger index can equivalently be decomposed as:

$$ML_0^{t,t+1} = MLECH_t^{t+1} \cdot MLTCH_t^{t+1} \quad (14)$$

$MLECH_t^{t+1}$  and  $MLTCH_t^{t+1}$  denote efficiency change and technological change respectively, where

$$MLECH_0^{t,t+1} = \frac{1 + \bar{D}_0^t(x^t, y^t, b^t; y^t, -b^t)}{1 + \bar{D}_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}$$

$$MLTCH_0^{t,t+1} = \left[ \frac{\{1 + \bar{D}_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})\}}{\{1 + \bar{D}_0^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})\}} \times \frac{\{1 + \bar{D}_0^{t+1}(x^t, y^t, b^t; y^t, -b^t)\}}{\{1 + \bar{D}_0^t(x^t, y^t, b^t; y^t, -b^t)\}} \right]^{1/2}$$

Construction of the Malmquist–Luenberger index can be carried out by solving a linear program detailed in Appendix A. The distance function  $\hat{D}_0^t(x^{t+1}, y^{t+1}, b^{t+1}; y, -b)$  yields an infeasible solution if a set of inputs and outputs  $(x^{t+1}, y^{t+1}, b^{t+1})$  is outside the production set and the movement along the direction vector  $g$  does not intersect the production frontier (Figure 4). Construction of the productivity index requires the existence of four distance functions,  $\bar{D}_0^{t,t}, \bar{D}_0^{t+1,t+1}, \bar{D}_0^{t,t+1}, \bar{D}_0^{t+1,t}$ , which measure distance to the frontier. Here  $\bar{D}_0^{t,t+1}$  denotes the distance function under the frontier at  $t$  using  $t+1$  data, i.e.  $\bar{D}_0^t(x^{t+1}, y^{t+1}, b^{t+1})$ . Since the frontier expands as time goes on, we expect that the  $t+1$  data point would be outside the frontier of the previous year ( $t$ ). So, the inter-period distance function  $\bar{D}_0^{t,t+1}$  cannot be calculated if the movement along the direction vector  $g$  does not intersect the production frontier. This will happen if the data at  $t+1$  is located outside the current frontier. Therefore, productivity estimates may not be available for a country that is very innovative or has a unique input–output structure, so that the shape of the frontier near that country heavily depends on her performance.

The procedures outlined above provide us with index number approaches to point estimates of productivity growth and its decomposition. A reasonable criticism of the methodology so far discussed is that there is no inference possible since we have a nonparametric model that does not assume any form for the production function (it is estimated nonparametrically as a

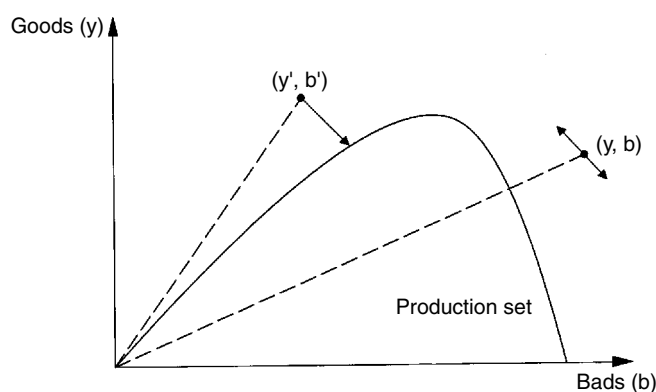


Figure 4. Mixed periods distance functions

piecewise linear convex function) and that no statistical model has been introduced. Results in the next section provide researchers with methods with which to construct significance bounds. Instead of relying on a simple point estimate for growth based on the index number, we can evaluate whether or not productivity growth and its decompositions are significantly different from zero.

### 3. BOOTSTRAPPING THE PRODUCTIVITY INDEX

The index numbers outlined above provide us with point estimates of productivity growth rates and the decompositions into their technical and efficiency components. Clearly, there is sampling variability and thus statistical uncertainty about these estimates. We address this issue by turning to economic theory (Debreu, 1951). We follow neoclassical theory and assume a data generating process (DGP) wherein firms randomly deviate from the underlying true frontier. Random deviations from the contemporaneous frontier at time  $t$  are measured by the distance function. The Simar and Wilson (2000a) bootstrapping method can be used to provide a statistical interpretation to the Malmquist/Malmquist–Luenberger index which has been used by Boyd *et al.* (1999).

Although the nonparametric programming estimators outlined above are taken to be deterministic, they measure performance relative to an estimate of the true and unobservable production frontier. Since estimates of the frontier are based on finite samples, efficiency and productivity measures based on these estimates are subject to sampling variation of the frontier. Simar and Wilson (1998, 2000a) outline a smooth bootstrap procedure to examine the sensitivity of distance functions, and hence efficiency, to sampling variation of the frontier.<sup>3</sup>

Extending these further, Simar and Wilson (1999) develop bootstrap methods to estimate the sampling distribution and confidence intervals for the Malmquist index which we extend here to the directed distance measure. We focus our discussion below on the bootstrapping algorithm for the standard Malmquist index for notational convenience. To anticipate results, the procedure yields bootstrap values  $\{\hat{M}_i^*(t, t+1)(b)_{b=1}^B\}$ ,  $\{\hat{E}_i^*(t, t+1)(b)_{b=1}^B\}$  and  $\{\hat{T}_i^*(t, t+1)(b)_{b=1}^B\}$ , where  $B$  is the number of bootstrap estimates. The bootstrap procedure then uses these values to approximate the unknown distribution and confidence interval of the original estimates. Thus,  $[\hat{M}_i(t, t+1) - M_i(t, t+1)] \overset{approx.}{\sim} [\hat{M}_i^*(t, t+1) - \hat{M}_i(t, t+1)|\ell]$  and  $prob(-b_\alpha \leq \hat{M}_i(t, t+1) - M_i(t, t+1) \leq -a_\alpha)$  is approximated by  $prob(-b_\alpha^* \leq \hat{M}_i^*(t, t+1) - \hat{M}_i(t, t+1) \leq -a_\alpha^*|\ell)$ , where  $\ell$  is the original data and  $\alpha$  is set to the desired confidence level.

<sup>3</sup> Simar and Wilson (2000b) summarize asymptotic results for efficiency estimates based on the DEA approach. Given certain regularity assumptions for the DGP, the DEA efficiency estimator is consistent for the univariate case,  $p = 1$  and  $q \geq 1$ , where  $p$  and  $q$  are the number of inputs and outputs respectively. This result comes from Banker (1993), who proves weak consistency for the DEA efficiency estimator where  $\hat{\theta}_{DEA(x,y)} \xrightarrow{p} \theta(x, y)$ . For the multivariate case,  $p \geq 1$  and  $q \geq 1$ , Kneip *et al.* (1998, 2001) provide results on the consistency of DEA as well as its rate of convergence. They prove that  $\hat{\theta}_{DEA(x,y)} - \theta(x, y) = O_p(n^{-(2/p+q+1)})$ . These results highlight the curse of dimensionality, where convergence of the efficiency estimator will suffer for large  $p$  and  $q$ . In addition to consistency, asymptotic results on the sampling distributions of DEA estimators are provided by Gijbels *et al.* (1999) for the univariate case, where  $p = q = 1$ . They demonstrate that  $\hat{\theta}_{DEA(x,y)} - \theta(x, y) \overset{asympt.}{\sim} F(\cdot, \cdot)$ , where  $F$  is a regular distribution function defined up to some unknown constants. However, these results have some drawbacks. They include asymptotic results, which may be misleading in small-sample contexts, the introduction of additional noise in estimating the unknown parameters of the limiting distribution, and the availability of only univariate results for the DEA estimators of efficiency. These limitations make the bootstrap an attractive alternative.

In general, bootstrapping involves simulating the data generating process and applying the estimator to each simulated sample in order to mimic the sampling distribution of the original estimator. The naive bootstrap involves estimation of  $f(x, y)$ , the distribution of the input–output pair, by the empirical distribution of the observed sample, through resampling with replacement. However, this yields inconsistent bootstrap approximations since empirical distributions are used to estimate distribution functions with bounded and unknown support on the production set. Specifically, by placing a positive probability mass at the boundary of the estimated production set, the empirical distribution gives inconsistent estimates of the underlying efficiency measures. Therefore, the key behind bootstrapping in this case lies in simulating the DGP appropriately. This can be done by using a smooth bootstrap.

For panel data, with the possibility of temporal correlation in the data, this can be accomplished by using a bivariate kernel estimator of the joint density of the original distance function estimates,  $\{\hat{D}_o^t(y_{it}, x_{it}), \hat{D}_o^{t+1}(y_{it+1}, x_{it+1})\}_{i=1}^N$ . Such an estimator is given by  $\hat{f}(z) = [1/(Nh^2)] \sum_{i=1}^N k((z - z_i)/h)$  where  $z$  is  $(1 \times 2)$ ,  $z_i = [\hat{D}_o^t(y_{it}, x_{it}), \hat{D}_o^{t+1}(y_{it+1}, x_{it+1})]$  is the  $i$ th row of the  $(N \times 2)$  matrix of the original distance function estimates,  $h$  is bandwidth and  $k(\cdot)$  is the bivariate kernel function. Since  $\{\hat{D}_o^t(y_{it}, x_{it}), \hat{D}_o^{t+1}(y_{it+1}, x_{it+1}), i = 1, \dots, N\}$ , is bounded from below by unity, the support of  $f(z)$  is bounded and hence the density estimated by  $\hat{f}(z)$  is inconsistent and asymptotically biased. Using the reflection method proposed by Silverman (1986) overcomes this problem. This involves using kernel methods to estimate the density of the original observations and their reflections about the boundaries, which is unity, in two-dimensional space. Details of the nine-step bootstrapping algorithm are provided in Appendix B.

We form confidence intervals for each index by sorting the bootstrap values in ascending order, deleting  $(\alpha/2)$  of the elements at either end and setting  $-b_\alpha^*$  and  $-a_\alpha^*$  equal to the end-points of the resulting sorted vector. This yields an estimated  $(1 - \alpha)\%$  confidence interval,  $\hat{M}_i(t, t + 1) + a_\alpha^* \leq M_i(t, t + 1) \leq \hat{M}_i(t, t + 1) + b_\alpha^*$ , for the Malmquist index. Confidence intervals for its components are obtained similarly. The estimated index is statistically significantly different from unity if the interval does not contain one.

We next turn to how these econometric developments may affect our comfort in and use of productivity forecasts for growth in the USA, OECD and in Asia, in the international debate on trade-offs between growth and environmental protection.

#### 4. ANALYSIS OF PRODUCTIVITY GROWTH CONTROLLING FOR CO<sub>2</sub> EMISSION

We calculate productivity growth and its components from a sample of 17 OECD countries during the period 1980–1990 using the data from the Penn World Tables (Mark 5.6) and the US Energy Information Administration. We then examine similar measures for a sample of 11 Asian countries during the period 1980–1995. Our measure of aggregate outputs are gross domestic product (GDP) as the desirable output and carbon dioxide (CO<sub>2</sub>) emission from the combustion of energy as the undesirable output. Capital stock, employment and energy are aggregate input proxies. GDP and capital stock are measured in 1985 international prices. Employment is calculated from real GDP per worker and capital is obtained from capital stock per worker. CO<sub>2</sub> emission accounts for only the combustion of energy. Although the Kyoto accords suggested establishment of a market in CO<sub>2</sub> emission credits for OECD countries, it exempted developing countries such as those in Asia, e.g. China.

#### 4.1. OECD Country Results

We estimate three types of Malmquist productivity indices and the Malmquist–Luenberger index. We use the terminology of Boyd *et al.* (1999). The first index is labelled Standard and ignores carbon dioxide completely. This is the traditional index calculated in the productivity growth literature. The second is labelled More Outputs and recognizes the jointness of the aggregate production frontier in output and in carbon dioxide but does nothing to account for the deleterious aspect of CO<sub>2</sub> production. The third is labelled More Goods and holds CO<sub>2</sub> emissions constant between the two periods of comparison and allows the level of good outputs to increase. This is a direction that seems most in agreement with the goals of the Kyoto protocols. The fourth is the Malmquist–Luenberger index and reduces CO<sub>2</sub> emissions between the two periods by the same proportion that GDP is allowed to increase. This direction can be viewed as a compromise between the goals of the pro-growth and anti-growth environmental movements. Table I lists the output and input growth rates for the OECD countries. A summary of productivity changes for each of the 17 countries, based on the four scenarios for treating environmental factors in the growth accounting exercise, are tabulated in Tables II, III and IV. The results suggest that there has been improvement in productivity due largely to technical change.

CO<sub>2</sub> emissions account for over 80% of total greenhouse gas emissions. When we account for the effect of CO<sub>2</sub> emissions on productivity growth (More Goods) we find marginally higher productivity growth rates. Table II shows total factor productivity growth rates under various assumptions on CO<sub>2</sub> emissions. Since the distance function measures the distance to the production frontier using a particular direction vector, total factor productivity growth depends on both the change in a country's input–output combination and the shape of the production frontier in the neighbourhood of that country's input–output observation. When frontier production of desirable outputs increases when production of undesirable (CO<sub>2</sub>) output increases, total factor productivity growth (TFP) depends on the change in CO<sub>2</sub> emissions. If CO<sub>2</sub> emissions are constrained not to increase in the calculation of productivity growth (the More Goods case), then we would expect a higher TFP growth rate for the country that reduces CO<sub>2</sub> emissions over the sample

Table I. Average growth rate of inputs and outputs (%) (1980–1990)

	GDP	Carbon dioxide	Capital	Labour	Energy
Australia	2.96	3.18	3.91	1.88	3.01
Austria	2.12	0.45	3.93	0.83	0.79
Belgium	1.89	−0.90	2.12	0.54	−0.05
Canada	2.97	0.15	5.17	1.14	1.34
Denmark	2.10	−1.84	2.21	0.58	−0.60
Finland	3.05	−0.45	3.91	0.73	1.52
France	2.22	−2.69	3.01	0.96	0.35
Germany	2.14	−0.94	2.73	1.34	0.18
Greece	1.87	4.01	2.21	0.53	3.29
Ireland	3.43	1.21	2.83	0.69	1.46
Italy	2.14	1.01	2.99	0.75	1.28
Japan	4.17	0.51	5.99	0.80	1.75
Norway	2.43	0.22	2.52	0.92	1.66
Spain	3.05	0.51	4.44	0.95	1.76
Sweden	2.01	−4.71	3.89	0.68	0.38
UK	2.85	0.03	3.01	0.51	0.72
USA	2.64	0.43	3.49	1.13	1.05

Table II. Comparison of average annual productivity growth (1980–1990)

Country	Malmquist			Malmquist–Luenberger
	Standard	More Outputs <sup>a</sup>	More Goods <sup>b</sup>	Standard <sup>c</sup>
Australia	1.0078	1.0048	n.a	n.a
Austria	1.0104	1.0051	1.0147	1.0067
Belgium	1.0142	1.0061	1.0136	1.0127
Canada	1.0181	1.0155	1.0203	1.0157
Denmark	1.0212	0.9970	1.0153	n.a
Finland	1.0207	1.0137	1.0244	1.0202
France	1.0133	1.0133	1.0216	1.0148
Germany	1.0100	1.0061	1.0079	1.0087
Greece	0.9946	1.0092	1.0099	0.9950
Ireland	1.0086	1.0029	1.0060	1.0060
Italy	1.0069	1.0053	1.0062	1.0072
Japan	1.0238	1.0096	1.0330	1.0181
Norway	1.0154	1.0154	1.0207	1.0136
Spain	0.9988	0.9975	0.9979	1.0078
Sweden	1.0135	1.0135	1.0347	1.0247
UK	1.0065	1.0095	n.a	n.a
USA	1.0080	0.9907	n.a	n.a
Average	1.0113 <sup>d</sup>	1.0068	1.0162	1.0116

<sup>a</sup> USA for 1980–1987, 1988–1990.

<sup>b</sup> Belgium for 1980–1981, 1982–1990; Denmark for 1988–1990; Greece for 1980–1983, 1985–1986.

<sup>c</sup> Ireland for 1980–1987; Italy for 1983–1990; Spain for 1982–1990.

<sup>d</sup> 1.0014 for the countries which correspond to the Malmquist–Luenberger indices.

Table III. Comparison of average annual efficiency change growth (1980–1990)

Country	Malmquist			Malmquist–Luenberger
	Standard	More Outputs <sup>a</sup>	More Goods <sup>b</sup>	Standard <sup>c</sup>
Australia	0.9946	1.0000	n.a	n.a
Austria	0.9993	0.9989	0.9994	0.9965
Belgium	1.0003	0.9986	0.9934	1.0005
Canada	1.0031	1.0031	1.0040	1.0033
Denmark	1.0097	1.0000	1.0000	n.a
Finland	1.0075	1.0022	1.0097	1.0082
France	1.0001	1.0001	1.0027	1.0024
Germany	0.9964	0.9963	0.9950	0.9955
Greece	0.9883	1.0017	0.9923	0.9901
Ireland	1.0080	1.0000	1.0001	1.0118
Italy	1.0000	1.0000	1.0000	1.0000
Japan	1.0152	1.0036	1.0152	1.0096
Norway	0.9996	0.9996	1.0009	0.9985
Spain	0.9965	0.9961	0.9974	1.0023
Sweden	0.9997	0.9997	1.0104	1.0090
UK	1.0000	1.0256	n.a	1.0000
USA	1.0000	1.0000	n.a	1.0000
Average	1.0011	1.0015	1.0015	1.0021

<sup>a</sup> USA for 1980–1987, 1988–1990.

<sup>b</sup> Belgium for 1980–1981, 1982–1990; Denmark for 1988–1990; Greece for 1980–1983, 1985–1986.

<sup>c</sup> Ireland for 1980–1987; Italy for 1983–1990; Spain for 1982–1990.

Table IV. Comparison of average annual technical change growth (1980–1990)

Country	Malmquist			Malmquist–Luenberger
	Standard	More Outputs <sup>a</sup>	More Goods <sup>b</sup>	Standard <sup>c</sup>
Australia	1.0132	1.0048	n.a	n.a
Austria	1.0111	1.0063	1.0153	1.0102
Belgium	1.0140	1.0075	1.0203	1.0122
Canada	1.0150	1.0123	1.0162	1.0124
Denmark	1.0113	0.9970	1.0153	n.a
Finland	1.0131	1.0115	1.0145	1.0119
France	1.0133	1.0133	1.0188	1.0124
Germany	1.0137	1.0098	1.0130	1.0132
Greece	1.0064	1.0075	1.0178	1.0049
Ireland	1.0006	1.0029	1.0059	0.9943
Italy	1.0069	1.0053	1.0062	1.0072
Japan	1.0085	1.0060	1.0175	1.0083
Norway	1.0158	1.0158	1.0198	1.0151
Spain	1.0023	1.0014	1.0005	1.0054
Sweden	1.0138	1.0138	1.0241	1.0156
UK	1.0065	0.9843	n.a	n.a
USA	1.0080	0.9907	n.a	n.a
Average	1.0102	1.0053	1.0146	1.0095

<sup>a</sup> USA for 1980–1987, 1988–1990.

<sup>b</sup> Belgium for 1980–1981, 1982–1990; Denmark for 1988–1990; Greece for 1980–1983, 1985–1986.

<sup>c</sup> Ireland for 1980–1987; Italy for 1983–1990; Spain for 1982–1990.

period. Finland, France and Sweden show improved TFP growth rates because they decreased CO<sub>2</sub> emissions. Countries whose CO<sub>2</sub> emissions increased, such as Ireland, Italy and Spain, have lower productivity growth rates. If the change in CO<sub>2</sub> emissions is small relative to TFP growth (in our sample less than 1% per year) the effects are reversed. This occurs in Austria, Belgium, Canada, Germany, Japan and Norway. Denmark and Greece show unexpected movements with relatively larger changes in CO<sub>2</sub> emissions. When the production frontier does not expand towards the GDP direction when CO<sub>2</sub> emissions increase then this can occur. It can also occur when the sample size is small. Though we recognize the negative effect of CO<sub>2</sub> emissions and assume that the desirable direction is not to increase CO<sub>2</sub> emissions, the positive trend of the countries' TFP growth rates does not change with the modification of the direction vector.

The results of Table II might be viewed as counterintuitive. For example, Canada had positive CO<sub>2</sub> growth and might seem to imply that the More Goods approach (which constrains CO<sub>2</sub>) should have less output and therefore less productivity growth. However, the results for More Growth indicate a higher productivity growth rate. The reason is that the productivity measure depends on the change of a country's relative location on the production set, in other words, the change of the distance between the frontier and the data. Thus productivity depends on the change of each country's performance and the frontier. Therefore, we cannot conjecture the size of the productivity growth by looking at country data only. Positive CO<sub>2</sub> growth of Canada does not imply low productivity growth since the frontier also changes. Only relative change matters.

The Malmquist–Luenberger productivity index imposes a more strict restriction on CO<sub>2</sub> outputs and is consistent with concerns of global warming. It assumes that an expansion of goods and

a reduction of bads is the desirable direction of development. As CO<sub>2</sub> emissions change, we expect that TFP growth rates will move in the opposite direction of the trend in CO<sub>2</sub> emissions. France and Sweden indicate higher growth rates because of the reductions of CO<sub>2</sub> emissions. Austria, Canada, Ireland, Japan and Norway are the countries that increase CO<sub>2</sub> emissions, so they show slower TFP growth rates than the standard Malmquist productivity index which ignores CO<sub>2</sub> emissions. This result is consistent with the findings of Ball *et al.* (2001) and Boyd *et al.* (1999). The rest of the countries do not show the inverse relationship between the change in CO<sub>2</sub> emissions and TFP growth rates because of the shape of their production frontier. Greece shows a slight increase in its TFP growth rate, though CO<sub>2</sub> emissions increased by 4.01% per year. This can be explained in part by the location of Greece's output mix. The carbon intensity of Greece, 0.3260 carbon ton/\$1000, is well outside the interval in which most countries lie (Table V). We have few data around Greece and the frontier does not expand towards the GDP direction with the increase of CO<sub>2</sub> emission. Though we use a directional distance function to address the negative effect of CO<sub>2</sub> emissions, most OECD countries still show positive TFP growth rates. OECD countries accomplish their growth in a lesser carbon-emitting way. This can be verified from the trend of carbon intensity. This result is consistent with the findings of Ball *et al.* (2001) and Boyd *et al.* (1999).

Table V reports the trends in each member country's carbon intensities during the sample period and shows significant improvement made by the OECD during the 1980s. This result may be explained in part by environmental regulations in place in the member countries which are intended to reduce sulphur dioxide and nitrogen dioxide emissions because of public health concerns. Policies that reduce sulphur dioxide or nitrogen dioxide play a complementary role in reducing carbon dioxide emission.

The indices are point estimates and an innovation of this paper is to provide a statistical interpretation to the index number measures. In order to bound them with a confidence interval we

Table V. The trend of carbon intensity (ton/1985 \$1000)

	1980	1985	1990	AAGR (%)
Australia	0.2950	0.2868	0.3014	0.2
Austria	0.2084	0.1898	0.1767	-1.6
Belgium	0.3422	0.2884	0.2593	-2.7
Canada	0.3683	0.3026	0.2790	-2.7
Denmark	0.3179	0.2672	0.2146	-3.9
Finland	0.3013	0.2231	0.2131	-3.4
France	0.2138	0.1602	0.1307	-4.8
Germany	0.2831	0.2463	0.2084	-3.0
Greece	0.2650	0.2791	0.3260	2.1
Ireland	0.2655	0.2181	0.2140	-2.1
Italy	0.1758	0.1663	0.1573	-1.1
Japan	0.2214	0.1723	0.1548	-3.5
Norway	0.1864	0.1501	0.1497	-2.2
Spain	0.2136	0.2051	0.1664	-2.5
Sweden	0.2329	0.1557	0.1179	-6.6
UK	0.2929	0.2527	0.2217	-2.7
USA	0.3705	0.3146	0.2981	-2.2

Note: AAGR means average annual growth rate.

turn to the bootstrapping procedures discussed above.<sup>4</sup> We use the original estimator in constructing the confidence interval of the true index. Based on the Malmquist–Luenberger index, in the 1980s (Table VI) confidence intervals derived from the bootstrap show that there is significant aggregate productivity change for most countries. However, we cannot tell whether efficiency change or technological change drives this productivity change. The disaggregated indices do not show statistically significant change (Tables VII and VIII).<sup>5</sup>

Table VI. Changes in Malmquist–Luenberger productivity index (CRS)

	1980–81	1981–82	1982–83	1983–84	1984–85	1985–86	1986–87	1987–88	1988–89	1989–90
Austria	0.9931	1.0351*	1.0312*	0.9685*	1.0094*	0.9852*	1.0126	1.0167	1.0192*	0.9978
Belgium	0.9756*	1.0187*	1.0123	1.0116*	0.9970	1.0091*	1.0193*	1.0431*	1.0220*	1.0194*
Canada	1.0340*	0.9644	1.0225*	1.0388*	1.0244*	1.0308*	1.0217*	1.0177	1.0081	0.9965*
Finland	1.0082	1.0435*	1.0297*	1.0229*	0.9929*	1.0070*	1.0246*	1.0454*	1.0450*	0.9847*
France	1.0093	1.0174*	1.0014	1.0131*	1.0062*	1.0397*	1.0139*	1.0449*	0.9984	1.0044
Germany	0.9829*	0.9759*	1.0198*	1.0241*	1.0036	1.0131*	1.0140*	1.0294*	0.9960	1.0295*
Greece	1.0022	1.0205	0.9753*	0.9963	0.9874*	0.9957	0.9704	0.9988	1.0162*	0.9881
Ireland	1.0035	0.9940	0.9719*	1.0386	0.9677*	0.9603*	1.1144*	n.a	n.a	n.a
Italy	n.a	n.a	n.a	0.9867*	1.0022	1.0078	1.0080*	0.9820	1.0207	1.0446*
Japan	1.0262*	1.0509*	1.0283*	0.9834*	1.0335*	1.0273*	1.0140*	0.9994	1.0079*	1.0112*
Norway	1.0527*	1.0291*	1.0168*	1.0213*	1.0101	1.0085	0.9890*	1.0139	0.9802*	1.0164*
Spain	n.a	n.a	0.9936	1.0012	1.0105	1.0159*	1.0293*	1.0172*	0.9800*	1.0153
Sweden	1.0119	1.0606*	1.0395*	1.0386*	0.9866*	1.0156*	1.0159*	1.0173*	1.0446*	1.0186*

Note: \* Denotes significant differences from unity at 0.05.

Table VII. Changes in efficiency (CRS)

	1980–81	1981–82	1982–83	1983–84	1984–85	1985–86	1986–87	1987–88	1988–89	1989–90
Austria	0.9982	1.0239	1.0170	0.9546*	1.0102	0.9539*	1.0140	0.9772	1.0266	0.9922
Belgium	0.9687	1.0290	1.0001	0.9883	0.9823	0.9914	1.0092	1.0179	1.0065	1.0126
Canada	1.0228	0.9762	1.0073	1.0031	1.0067	1.0153	1.0156	1.0019	0.9920	0.9925
Finland	1.0161	1.0357*	1.0196	1.0077	0.9874	0.9862	1.0140	1.0129	1.0296*	0.9741*
France	1.0162	1.0078	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Germany	0.9761	0.9997	1.0020	0.9942	0.9882	0.9938	1.0023	1.0010	0.9805	1.0179
Greece	1.0079	1.0110	0.9628*	0.9869	0.9879	0.9765	0.9779	0.9801	1.0219	0.9900
Ireland	1.0298	0.9967	0.9684	1.0484	0.9545*	0.9476*	1.1515*	n.a	n.a	n.a
Italy	n.a	n.a	n.a	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Japan	1.0316	1.0388*	1.0126	0.9719	1.0340	1.0016	1.0207	0.9735	1.0157	0.9985
Norway	1.0291	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9884	0.9654*	1.0036
Spain	n.a	n.a	0.9865	0.9900	1.0088	1.0019	1.0334	0.9938	0.9869	1.0184
Sweden	1.0177	1.0513*	1.0224	1.0000	1.0000	0.9838	1.0019	0.9740	1.0415	1.0000

<sup>4</sup> The bootstrap results indicate that the corrected estimator has the higher mean squared error:

$$\text{var}\{\widehat{ML}_0^{t,t+1}(p)\} > \frac{1}{3}(\widehat{bias}_B[\widehat{ML}_0^{t,t+1}])^2$$

<sup>5</sup> There is no guarantee that the solutions from the programming problems yield a feasible solution and in fact this happens in several countries. In this case the productivity index is not available (n.a.).

Table VIII. Changes in technology (CRS)

	1980–81	1981–82	1982–83	1983–84	1984–85	1985–86	1986–87	1987–88	1988–89	1989–90
Austria	0.9949	1.0109	1.0139	1.0145	0.9991	1.0328*	0.9986	1.0405	0.9928	1.0056
Belgium	1.0071	0.9900	1.0122	1.0236	1.0150	1.0179	1.0101	1.0247	1.0155	1.0067
Canada	1.0109	0.9879	1.0151	1.0356	1.0176	1.0153	1.0060	1.0158	1.0162	1.0041
Finland	0.9923	1.0075	1.0099	1.0150	1.0056	1.0211	1.0104	1.0321*	1.0150	1.0109
France	0.9933	1.0095	1.0014	1.0131	1.0062	1.0397*	1.0139	1.0449	0.9984	1.0044
Germany	1.0069	0.9762	1.0177	1.0301	1.0155	1.0194	1.0117	1.0284	1.0158	1.0114
Greece	0.9944	1.0094	1.0130	1.0095	0.9995	1.0196	0.9923	1.0191	0.9945	0.9981
Ireland	0.9744	0.9973	1.0037	0.9906	1.0138	1.0134	0.9677*	n.a	n.a	n.a
Italy	n.a	n.a	n.a	0.9867	1.0022	1.0078	1.0080	0.9820	1.0207	1.0446
Japan	0.9948	1.0117	1.0154	1.0119	0.9995	1.0257*	0.9935	1.0265	0.9924	1.0128
Norway	1.0230	1.0291	1.0168	1.0213	1.0101	1.0085	0.9890	1.0258	1.0153	1.0128
Spain	n.a	n.a	1.0072	1.0113	1.0018	1.0140	0.9961	1.0235	0.9930	0.9970
Sweden	0.9944	1.0089	1.0166	1.0386*	0.9866	1.0323*	1.0140	1.0444	1.0029	1.0186

#### 4.2. Asian Results

We next turn to our analysis of Asia. The countries are: China, Hong Kong, India, Indonesia, Japan, Korea, Malaysia, Singapore, Philippines, Taiwan, and Thailand. Aggregate country data are from the Penn World Tables (Mark 5.6) and the International Monetary Fund's International Financial Statistics. The CO<sub>2</sub> emission data come from the US Energy Information Administration.

Table IX shows the output and input growth rates for the sample of Asian countries. Tables X, XI and XII provide results analogous to those contained in Tables II, III and IV for the OECD countries. If carbon dioxide emissions are ignored, productivity growth can be found only in Japan, Korea, Taiwan, Singapore and Hong Kong. This is consistent with the findings of Young (1995), who pointed out that the bulk of post-WWII growth in Asian countries was due to input growth and not TFP growth.

When we apply the directional distance function methods, Japan is the only country that shows positive productivity growth over the entire sample period. This is consistent with the other OECD countries. No productivity growth now can be found for Korea, Taiwan, Singapore and Hong Kong, countries that showed positive TFP growth without the consideration of CO<sub>2</sub>. This may indicate that measured TFP growth in these countries was distorted by a failure to properly account for the growth in environmental bads.

Table IX. Average growth rate of inputs and outputs (1980–1995)

	GDP	Carbon dioxide	Capital	Labour	Energy
China	6.50	4.79	7.64	2.06	5.11
Hong Kong	6.40	5.27	7.89	1.72	5.89
India	5.64	6.90	5.58	1.84	6.77
Indonesia	6.36	6.21	10.67	2.29	7.59
Japan	3.31	0.50	5.12	0.65	2.09
Korea	8.70	7.41	11.99	2.03	9.23
Malaysia	6.71	8.04	9.74	2.96	8.79
Philippines	1.94	2.51	3.96	2.64	3.40
Singapore	7.45	6.05	8.58	2.25	6.80
Taiwan	7.61	5.71	7.52	1.89	6.49
Thailand	7.33	10.21	9.79	2.00	10.33

Table X. Comparison of average annual productivity growth (1980–1995)

Country	Malmquist			Malmquist–Luenberger <sup>b</sup>
	Standard	More Outputs	More Goods <sup>a</sup>	
China	0.9952 <sup>c</sup>	0.9875	n.a	1.0079
Hong Kong	1.0147	1.0083	1.1099	n.a
India	0.9998	1.0020	0.9996	n.a
Indonesia	0.9736	0.9759	0.9701	0.9847
Japan	1.0254	1.0246	1.0268	1.0259
Korea	1.0048	0.9808	0.9897	0.9921
Malaysia	0.9749	0.9922	0.9778	0.9964
Philippines	0.9819	0.9823	0.9874	0.9934
Singapore	1.0446	1.0146	n.a	n.a
Taiwan	1.0046	0.9965	0.9960	0.9957
Thailand	0.9759	0.9785	0.9785	0.9744
Average	0.9996 <sup>d</sup>	0.9948	1.0040	0.9963

<sup>a</sup> Korea for 1980–1981, 1983–1995; Hong Kong for 1980–1982.

<sup>b</sup> China for 1989–1995; Korea for 1980–1984, 1985–1995; Taiwan for 1980–1994; Thailand for 1981–1982, 1983–1995; Indonesia for 1983–1995.

<sup>c</sup> 1.0087 for 1989–1995, which corresponds to the period of Malmquist–Luenberger.

<sup>d</sup> 0.9937 for eight countries which correspond to the Malmquist–Luenberger indices.

Table XI. Comparison of average annual efficiency change growth (1980–1995)

Country	Malmquist			Malmquist–Luenberger
	Standard	More Outputs	More Goods <sup>a</sup>	
China	1.0046	1.0000	n.a	1.0146
Hong Kong	1.0000	1.0000	1.0000	n.a
India	1.0116	1.0048	1.0097	n.a
Indonesia	0.9882	0.9926	0.9937	0.9963
Japan	0.9894	0.9894	0.9906	0.9913
Korea	1.0005	0.9908	0.9939	0.9970
Malaysia	0.9865	0.9991	0.9824	0.9916
Philippines	0.9934	0.9934	0.9964	0.9986
Singapore	1.0000	1.0000	n.a	n.a
Taiwan	1.0147	1.0039	1.0057	1.0124
Thailand	0.9881	0.9960	0.9885	0.9906
Average	0.9979	0.9973	0.9957	0.9991

<sup>a</sup> Korea for 1980–1981, 1983–1995; Hong Kong for 1980–1982.

<sup>b</sup> China for 1989–1995; Korea for 1980–1984, 1985–1995; Taiwan for 1980–1994; Thailand for 1981–1982, 1983–1995; Indonesia for 1983–1995.

The developing countries are arguably less interested in and well-equipped to handle waste byproducts in pursuing their economic policy. Confidence intervals derived from the bootstrap show that there is also significant aggregate productivity change for most Asian countries (Table XIII). However, we cannot tell whether efficiency change or technological change drives this productivity change. The disaggregated indices do not show statistically significant change (Tables XIV and XV).

Table XII. Comparison of average annual technical change growth (1980–1995)

Country	Malmquist			Malmquist–Luenberger
	Standard	More Outputs	More Goods <sup>a</sup>	Standard <sup>b</sup>
China	0.9907	0.9875	n.a	0.9935
Hong Kong	1.0147	1.0083	1.1099	n.a
India	0.9884	0.9972	0.9900	n.a
Indonesia	0.9852	0.9832	0.9762	0.9884
Japan	1.0363	1.0356	1.0365	1.0352
Korea	1.0043	0.9899	0.9958	0.9968
Malaysia	0.9883	0.9931	0.9953	1.0048
Philippines	0.9884	0.9888	0.9909	0.9950
Singapore	1.0446	1.0146	n.a	n.a
Taiwan	0.9900	0.9927	0.9903	0.9837
Thailand	0.9877	0.9825	0.9898	0.9841
Average	1.0016	0.9976	1.0083	0.9977

<sup>a</sup> Korea for 1980–1981, 1983–1995; Hong Kong for 1980–1982.

<sup>b</sup> China for 1989–1995; Korea for 1980–1984, 1985–1995; Taiwan for 1980–1994; Thailand for 1981–1982, 1983–1995; Indonesia for 1983–1995.

Table XIII. Changes in Malmquist–Luenberger productivity index (CRS)

	China	Indonesia	Japan	Korea	Malaysia	Philippines	Taiwan	Thailand
1980–81	n.a	n.a	1.0443*	0.9441*	0.9984	1.0322*	0.8703*	n.a
1981–82	n.a	n.a	1.0248*	0.9518	1.0043	1.0187*	0.9780*	1.0203*
1982–83	n.a	n.a	1.0190*	0.9531*	0.9419*	0.9915	0.9934*	n.a
1983–84	n.a	0.9300*	1.0248*	0.9722*	0.9744	0.9887	0.9967*	0.9150*
1984–85	n.a	0.9355*	1.0426*	n.a	0.9623	1.0012	0.8828*	0.9119*
1985–86	n.a	1.0212*	1.0174*	1.0031	0.9603*	0.9995	1.1614*	1.0114*
1986–87	n.a	0.9681*	1.0345*	1.0358*	1.0319*	0.9585*	1.0639*	1.0046
1987–88	n.a	0.9968	1.0472*	1.0018	1.0454*	1.0073*	0.9861*	0.9852*
1988–89	n.a	0.9785*	1.0386*	1.0204*	1.0038	0.9719*	1.0047*	0.9670*
1989–90	0.9749	1.0250*	1.0416*	1.0095	0.9655*	1.0044*	0.9341*	0.9780*
1990–91	1.0001*	1.0025	1.0407*	0.9840*	1.0151	0.9836*	1.0866*	0.9686*
1991–92	1.0186*	0.9690*	1.0014*	1.0006	1.0082*	0.9647*	0.9733*	0.9830
1992–93	1.0224	0.9681*	1.0073*	0.9319*	0.9743*	0.9628*	1.0662*	0.9756*
1993–94	1.0173	1.0303*	0.9905	1.0323*	1.0224*	1.0106*	0.9425*	1.0014
1994–95	1.0151	0.9905	1.0144*	1.0488*	1.0371*	1.0054*	n.a	0.9449*

Note: \* Denotes significant differences from unity at 0.05.

Table XVI shows each country's efficient production combination in 1995, the end of the Asian sample. This is obtained using the Malmquist–Luenberger distance function and scaling radially its actual outputs to their frontier efficient levels. If the largest polluting country in Asia, China, could operate at her frontier, she could increase GDP and decrease carbon dioxide emission by 38% and attain a 0.146 ton/\$1000 carbon intensity. This is in line with the least polluting countries in the OECD (Table V).

### 4.3. Incremental Costs of Pollution Abatement

The choices of the direction vectors for CO<sub>2</sub> levels used in our analysis are consistent with current practices and proposed treatments of CO<sub>2</sub> in the discussions of world environmental policies. We

Table XIV. Changes in efficiency (CRS)

	China	Indonesia	Japan	Korea	Malaysia	Philippines	Taiwan	Thailand
1980–81	n.a	n.a	1.0000	1.0052	0.9818	1.0221	0.9112*	n.a
1981–82	n.a	n.a	1.0000	1.1144	1.0183	1.0022	1.0337*	1.0000
1982–83	n.a	n.a	1.0000	0.9873*	0.9581*	0.9829	1.0102	n.a
1983–84	n.a	0.9648	1.0000	0.9563	0.9674*	0.9989	0.9892	1.0000
1984–85	n.a	0.9905	1.0000	n.a	0.9651	1.0185	1.0333*	1.0000
1985–86	n.a	1.0082	1.0000	0.9208	0.9606	1.0000	1.0198	1.0000
1986–87	n.a	0.9894	0.9882	1.0255*	1.0221*	0.9939	1.0040	1.0000
1987–88	n.a	1.0009	0.9811	1.0067	1.0485*	1.0061	0.9865	1.0000
1988–89	n.a	1.0099	1.0162	1.0222	1.0070	1.0000	1.0613	1.0000
1989–90	0.9884	1.0378	1.0020	0.9784	0.9395*	1.0000	1.0072	1.0000
1990–91	1.0209	1.0000	0.9919	0.9791	1.0112	1.0000	1.1171*	1.0000
1991–92	1.0242	1.0000	0.9389*	0.9966	1.0067	1.0000	0.9878	0.9962
1992–93	1.0235	0.9986	1.0005	0.9364*	0.9820	1.0000	1.0123	0.9961
1993–94	1.0045	0.9750	0.9545	0.9962	0.9775	0.9587	1.0000	0.9479
1994–95	1.0264	0.9806	0.9946	1.0334	1.0288	0.9955	n.a	0.9372

Table XV. Changes in technology (CRS)

	China	Indonesia	Japan	Korea	Malaysia	Philippines	Taiwan	Thailand
1980–81	n.a	n.a	1.0443	0.9393	1.0170	1.0099	0.9551	n.a
1981–82	n.a	n.a	1.0248	0.8541	0.9862	1.0165	0.9462*	1.0203
1982–83	n.a	n.a	1.0190	0.9654*	0.9830	1.0087	0.9834*	n.a
1983–84	n.a	0.9649*	1.0248	1.0166	1.0072	0.9899	1.0076*	0.9150*
1984–85	n.a	0.9445*	1.0426	n.a	0.9971	0.9830	0.8543*	0.9119*
1985–86	n.a	1.0129	1.0174	1.0894	0.9996	0.9995	1.1388*	1.0114
1986–87	n.a	0.9784	1.0469	1.0101	1.0095	0.9644*	1.0597*	1.0046
1987–88	n.a	0.9959	1.0675*	0.9951	0.9970	1.0012	0.9996	0.9852
1988–89	n.a	0.9690	1.0221	0.9983	0.9968	0.9719	0.9466	0.9670
1989–90	0.9863	0.9876	1.0395	1.0318	1.0276	1.0044	0.9273*	0.9780
1990–91	0.9797	1.0025	1.0492	1.0050	1.0039	0.9836	0.9727	0.9686
1991–92	0.9945	0.9690	1.0655	1.0040	1.0015	0.9647*	0.9853	0.9868
1992–93	0.9989	0.9695	1.0068	0.9952	0.9921	0.9628*	1.0532*	0.9795
1993–94	1.0127	1.0567*	1.0377	1.0362	1.0460*	1.0542*	0.9425*	1.0565
1994–95	0.9890	1.0101	1.0199	1.0149	1.0080	1.0100	n.a	1.0082

Table XVI. Malmquist–Luenberger production frontier at 1995

Country	Actual value			Frontier		
	GDP	CO <sub>2</sub>	Intensity	GDP	CO <sub>2</sub>	Intensity
China	2452.2	792.3	0.323	3377.2	493.5	0.146
Hong Kong	111.3	12.1	0.109	111.3	12.1	0.109
India	1380.8	223.6	0.162	1380.8	223.6	0.162
Indonesia	479.4	57.3	0.119	502.1	54.5	0.109
Japan	1916.8	280.8	0.146	2193.2	240.3	0.110
Korea	412.3	102.3	0.248	551.5	67.8	0.123
Malaysia	138.5	23.3	0.168	168.3	18.3	0.109
Philippines	121.2	14.5	0.120	127.0	13.8	0.109
Singapore	47.3	21.0	0.445	47.3	21.0	0.445
Taiwan	238.5	46.7	0.196	238.5	46.7	0.196
Thailand	293.8	42.1	0.143	333.3	36.5	0.109

have examined growth accounting across the spectrum of direction vectors, from no constraints on pollution, to no increase over current levels, to a partial reduction, and have estimated productivity growth levels and their standard errors under these different scenarios. A final question we consider is what are the incremental costs of the various angles of the direction vector that directly determine the force of the constraint on the CO<sub>2</sub> levels that are not freely disposable as compared with the case in which no consideration is given to the environmental byproducts, that is they are assumed to be freely disposable. We adopt the following computational procedure. We first estimate the distance function under the set of restrictions on the bads considered above using the input–output data. Given the distance function, we then derive the production frontier using the specific restriction on CO<sub>2</sub> emissions. We calculate incremental costs by dividing the change in the frontier value of GDP under the different scenarios under the assumption of free disposability by the corresponding frontier level of CO<sub>2</sub> emissions. Calculation of the incremental costs of reductions in CO<sub>2</sub> emissions for nonfrontier observations is problematic. These calculations involve observations for which no constraint and hence no implicit valuation can be applied to incremental changes in inputs and/or outputs. Thus for countries that are not on the frontier, we must assume that the adjustment costs involving a movement towards the frontier are minimal. Since the bulk of the OECD countries appear to be near or at the frontier and few of the Asian countries are, we focus on the former set of countries. We estimate the median incremental cost of a reduction of CO<sub>2</sub> emission to be on the order of \$131/ton for the direction in which emission levels are maintained at their current levels and on the order of \$159/ton when we reduce CO<sub>2</sub> emissions by the projection of the ML index. These do not differ greatly, in part because for this exercise the countries are assumed to be at the frontier and then when technology displays little convexity the two treatments are quite similar. They also are estimated based on a technology that was in place in the 1980s. They are in the middle of the range of estimates for the carbon permit prices (c.f. Weyant and Hill, 1999; Nordhaus, 2001). There appears to be a strong positive relation between the incremental cost of CO<sub>2</sub> emissions and the carbon intensity of a country's economy, a result that has significant implications for developing economies whose carbon intensities are substantially larger than the OECD countries.

## 5. CONCLUSIONS

In this paper we have analysed the productivity growth of OECD and Asian countries, taking explicit account of environmental waste byproducts such as CO<sub>2</sub> which account for over 80% of total greenhouse gas emissions. The Malmquist–Luenberger productivity index is estimated to account for CO<sub>2</sub> and is compared with a reference Malmquist index that does not account for CO<sub>2</sub> emissions. When we include carbon dioxide as a bad output of the economies, average growth rates in total factor productivity for OECD countries show little change. Ball *et al.* (2001) and Boyd *et al.* (1999) found a marginal increase in TFP growth. Such marginally higher rates are found in our analysis when we only constrain levels of CO<sub>2</sub> to remain at sample levels (the More Goods case). The Asian economies on average show little apparent impact of such environmental accounting on their total factor productivity growth rates.

The confidence intervals derived by bootstrapping methods indicate that significant aggregate productivity growth in the Malmquist–Luenberger sense has taken place in the last decade in OECD. Asian countries showed significant negative productivity growth except Japan. This is consistent with the findings of Young (1995), who pointed out that the bulk of post-WWII

growth in Asian countries was due to input growth and not TFP growth. However, we cannot determine with nominal statistical confidence whether it is due to catching up (efficiency change) or innovation (technology change). We view the lack of significant results as some of the most important findings from our research. Index numbers are simply point estimates without any standard error. We have provided a methodology that can be used to further the debate on the effects of carbon taxes on productivity growth, whose statistical significance can be assessed via the bootstrapping algorithms.

#### APPENDIX A

The distance functions are constructed by the following programs:

$$\begin{aligned}
 (\hat{D}_0^t(x^t(k'), y^t(k'), b^t(k')))^{-1} &= \text{Max } \theta(k') \\
 \text{Subject to } \theta(k') y_m^t(k') &\leq \sum_{k=1}^K z^t(k) y_m^t(k) \quad m = 1, \dots, M \\
 \sum_{k=1}^K z^t(k) b_n^t(k) &= b_n^t(k') \quad n = 1, \dots, N \\
 \sum_{k=1}^K z^t(k) x_l^t(k) &\leq x_l^t(k') \quad l = 1, \dots, L \\
 z^t(k) &\geq 0 \quad k = 1, \dots, K
 \end{aligned}$$

This formulation represents a constant returns to scale technology whose inputs and desirable outputs are strongly disposable and whose undesirable outputs are weakly disposable. The constant returns to scale technology assumption can be relaxed to allow nonincreasing returns to scale or variable returns to scale. Those assumptions are applied by adding the restrictions  $\sum_{k=1}^K z^t(k) \leq 1$  or  $\sum_{k=1}^K z^t(k) = 1$ , respectively, instead of  $z^t(k) \geq 0$ .  $z^t(k)$  is an intensity variable indicating at what intensity a particular country's resources may be employed in production. The change from weak to strong disposability of undesirable outputs entails changing the equality of the second constraint to the inequality  $\sum_{k=1}^K z^t(k) b_n^t(k) \geq b_n^t(k')$ . Similarly, an inter-period distance function  $D_0^t(x^{t+1}, y^{t+1}, b^{t+1})$  can be constructed from the linear program:

$$\begin{aligned}
 (\hat{D}_0^t(x^{t+1}(k'), y^{t+1}(k'), b^{t+1}(k')))^{-1} &= \text{Max } \theta(k') \\
 \text{Subject to } \theta(k') y_m^{t+1}(k') &\leq \sum_{k=1}^K z^t(k) y_m^t(k) \quad m = 1, \dots, M \\
 \sum_{k=1}^K z^t(k) b_n^t(k) &= b_n^{t+1}(k') \quad n = 1, \dots, N \\
 \sum_{k=1}^K z^t(k) x_l^t(k) &\leq x_l^{t+1}(k') \quad l = 1, \dots, L \\
 z^t(k) &\geq 0 \quad k = 1, \dots, K
 \end{aligned}$$

Note that the reference technology is constructed from observations at  $t$ . Also  $(x^{t+1}, y^{t+1}, b^{t+1})$  need not belong to  $F^t$ , so  $\hat{D}_0^t(x^{t+1}(k'), y^{t+1}(k'), b^{t+1}(k'))$  can have values greater than 1.

Another type of Malmquist index can be defined by not differentiating between the desirable and undesirable outputs. The (more outputs) distance functions simply find a maximum possible production point along the radial hyperplane. They can be constructed by solving the linear program:

$$\begin{aligned} (\hat{D}_0^t(x^t(k'), y^t(k'), b^t(k')))^{-1} &= \text{Max } \theta(k') \\ \text{Subject to } \theta(k')y_m^t(k') &\leq \sum_{k=1}^K z^t(k)y_m^t(k) \quad m = 1, \dots, M \\ \sum_{k=1}^K z^t(k)b_n^t(k) &\geq \theta(k')b_n^t(k') \quad n = 1, \dots, N \\ \sum_{k=1}^K z^t(k)x_l^t(k) &\leq x_l^t(k') \quad l = 1, \dots, L \\ z^t(k) &\geq 0 \quad k = 1, \dots, K \end{aligned}$$

The Malmquist–Luenberger index can be constructed by solving the set of linear programming problems:

$$\begin{aligned} \hat{D}_0^t(x^{t+1}(k'), y^{t+1}(k'), b^{t+1}(k'); y^{t+1}(k'), -b^{t+1}(k')) &= \text{Max } \beta \\ \text{Subject to } (1 + \beta)y_m^{t+1}(k') &\leq \sum_{k=1}^K z^t(k)y_m^t(k) \quad m = 1, \dots, M \\ \sum_{k=1}^K z^t(k)b_n^t(k) &= (1 - \beta)b_n^{t+1}(k') \quad n = 1, \dots, N \\ \sum_{k=1}^K z^t(k)x_l^t(k) &\leq x_l^{t+1}(k') \quad l = 1, \dots, L \\ z^t(k) &\geq 0 \quad k = 1, \dots, K \end{aligned}$$

## APPENDIX B

The following algorithm is used to implement the density estimation of the original estimators and their reflections.

First, we form  $(N \times 1)$  vectors  $A = [\hat{D}_o^t(y_1, x_1) \dots \hat{D}_o^t(y_N, x_N)]'$  and  $B = [\hat{D}_o^{t+1}(y_1, x_1) \dots \hat{D}_o^{t+1}(y_N, x_N)]'$ .

Second, we reflect these values about the boundaries in two-dimensional space to form a  $(4N \times 2)$  matrix represented by:

$$\Delta = \begin{bmatrix} A & B \\ 2 - A & B \\ 2 - A & 2 - B \\ A & 2 - B \end{bmatrix}$$

The estimated covariance matrix of the columns of  $[A \ B]$ ,  $\widehat{\Sigma}$ , which is the same as that of the reflected data  $[2 - A \ 2 - B]$ , gives the temporal correlation of the original data. The covariance matrix of  $[2 - A \ B]$  and  $[A \ 2 - B]$  is given by  $\widehat{\Sigma}_R$ :

$$\widehat{\Sigma} = \begin{bmatrix} \widehat{\sigma}_1^2 & \widehat{\sigma}_{12} \\ \widehat{\sigma}_{12} & \widehat{\sigma}_2^2 \end{bmatrix} \quad \text{and} \quad \widehat{\Sigma}_R = \begin{bmatrix} \widehat{\sigma}_1^2 & -\widehat{\sigma}_{12} \\ -\widehat{\sigma}_{12} & \widehat{\sigma}_2^2 \end{bmatrix}$$

Third, we randomly draw with replacement  $N$  rows from  $\Delta$  to form an  $(N \times 2)$  matrix  $\Delta^* = [\delta_{ij}]$ ,  $i = 1, \dots, N$ ,  $j = 1, 2$ .

Fourth, we compute:

$$\bar{\delta}_{.j} = \frac{1}{N} \sum_{i=1}^N \delta_{ij}, \quad j = 1, 2$$

Fifth, we simulate draws from a bivariate  $N(0, \widehat{\Sigma})$  and  $N(0, \widehat{\Sigma}_R)$  by generating *iid* pseudo-random  $N(0, 1)$  deviates  $(z_1, z_2)$  s.t.  $(l_1z_1, l_2z_1 + l_3z_2) \sim N(0, \widehat{\Sigma})$  and  $(l_1z_1, -l_2z_1 + l_3z_2) \sim N(0, \widehat{\Sigma}_R)$ . Here,  $l_1, l_2, l_3$  are elements of a lower triangular matrix

$$L = \begin{bmatrix} l_1 & 0 \\ l_2 & l_3 \end{bmatrix}$$

obtained from the Cholesky decomposition of the  $(2 \times 2)$  matrix  $\widehat{\Sigma}$ . These simulated draws form  $e^*$ , an  $(N \times 2)$  matrix containing independent draws from the kernel functions. If  $\Delta_i^*$  is drawn from  $[A \ B]$  or  $[2 - A \ 2 - B]$ , the  $i$ th row of  $e^*$  is from  $N(0, \widehat{\Sigma})$ , but if  $\varepsilon^*$  is drawn from  $[2 - A \ B]$  or  $[A \ 2 - B]$ , the  $i$ th row of  $e^*$  is from  $N(0, \widehat{\Sigma}_R)$ .

Sixth, we compute an  $(N \times 2)$  matrix  $\Gamma$ :

$$\Gamma = (1 + h)^{-\frac{1}{2}} \left( \Delta^* + he^* - C \begin{bmatrix} \bar{\delta}_{.1} & 0 \\ 0 & \bar{\delta}_{.2} \end{bmatrix} \right) + C \begin{bmatrix} \bar{\delta}_{.1} & 0 \\ 0 & \bar{\delta}_{.2} \end{bmatrix}$$

where  $C$  is an  $(N \times 2)$  matrix of ones. Following the suggestion for bivariate data by Silverman (1986), the bandwidth  $h$  is set equal to  $(4/5N)^{1/6}$ .

Seventh, for each element of  $\gamma_{ij}$  of  $\Gamma$ , set

$$\gamma_{ij}^* = \begin{cases} \gamma_{ij} & \text{if } \gamma_{ij} \geq 1 \\ 2 - \gamma_{ij} & \text{otherwise} \end{cases}$$

The  $(N \times 2)$  matrix  $\Gamma^* = [\gamma_{ij}^*]$  contains simulated distance function values.

Eighth, using these we form pseudosamples  $\ell^*$  by setting  $x_{it_j}^* = x_{it_j}$  and  $y_{it_j}^* = [\widehat{D}_o^t(x_{it_j}, y_{it_j})y_{it_j}] / \gamma_{it_j}^*$ ,  $i = 1, \dots, N$ ,  $j = 1, 2$ . Since we are using output-oriented distance functions in our Malmquist computations, we calculate  $\widehat{D}_o^t(x_{it_j}, y_{it_j})y_{it_j}$  to scale the output vector to the estimated efficient frontier and divide it by  $\gamma_{it_j}^*$  to simulate a random deviation away from this frontier.

Ninth, we compute the four distance functions  $\widehat{D}_o^{*t}(x_{it}^*, y_{it}^*)$ ,  $\widehat{D}_o^{*t+1}(x_{it}^*, y_{it}^*)$ ,  $\widehat{D}_o^{*t}(x_{it+1}^*, y_{it+1}^*)$  and  $\widehat{D}_o^{*t+1}(x_{it+1}^*, y_{it+1}^*)$  on which the Malmquist TFP index and its components are based. We repeat steps three to nine  $B$  times to get a set of bootstrap estimates. We form confidence intervals for each index by sorting the bootstrap values in ascending order, deleting  $(\alpha/2)$  of the elements at either

end and setting  $-b_\alpha^*$  and  $-a_\alpha^*$  equal to the end-points of the resulting sorted vector. This yields an estimated  $(1 - \alpha)\%$  confidence interval,  $\hat{M}_i(t, t + 1) + a_\alpha^* \leq M_i(t, t + 1) \leq \hat{M}_i(t, t + 1) + b_\alpha^*$ , for the Malmquist index. Confidence intervals for its components are obtained similarly. The estimated index is statistically significantly different from unity if the interval does not contain one.

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